Exercise 7

Find the particular solution for each of the following initial value problems:

$$u' - u = 2xe^x, \quad u(0) = 0$$

Solution

This is an inhomogeneous first order linear ODE, so we can multiply both sides by the integrating factor,

$$I(x) = e^{\int (-1)\,dx} = e^{-x},$$

to solve it. The equation becomes

$$e^{-x}u' - e^{-x}u = 2x.$$

Observe that the left side can be written as $(e^{-x}u)'$ by the product rule.

$$\frac{d}{dx}(e^{-x}u) = 2x$$

Now integrate both sides with respect to x.

$$e^{-x}u = x^2 + C$$

The general solution is thus

$$u(x) = e^x(x^2 + C).$$

Because an initial condition is given, this constant of integration can be determined.

$$u(0) = e^0(0 + C) = C \quad \rightarrow \quad C = 0$$

Therefore,

$$u(x) = x^2e^x.$$